

First Applied Mathematics Symposium Münster

Variational Methods for Dynamic Inverse Problems and Imaging Schedule

All lectures will take place in lecture room **SRZ 19/20** of the lecture hall building Orléansring 12. The coffee breaks will be also there in the foyer of the building.

Please note: The distinguished lecture on Tuesday, September 29, with Stan Osher will take place in the castle building, Schlossplatz 2, in the lecture hall S 10 on the 4th floor.

Day 1 (Monday, September 28)

8:30 – 9:00	Registration
9:00 – 9:15	Opening <i>Chair: Martin Burger</i>
9:15 – 10:05	Bernadette Hahn (Saarland University, Saarbrücken): A time-dependent regularization method for motion compensation
10:05 – 10:35	Ozan Öktem (KTH, Stockholm): Shape based regularization
10:35 – 11:00	Coffee Break
11:00 – 11:30	<i>Chair: Jan-Frederik Pietschmann</i> Matthias Schlottbom (WWU Münster): Diffuse interface methods for inverse problems: Case study for an elliptic Cauchy problem
11:30 – 12:00	Martin Benning (Cambridge University): A preconditioned ADMM algorithm for convex optimization with nonlinear operator constraints
12:00 – 12:30	Romana Boiger (Alpen-Adria University, Klagenfurt): An online parameter identification method for time-dependent problems
12:30 – 13:00	Anar Rahimov (Marseille University): On numerical solution on inverse problems for a parabolic equation
13:00 – 14:30	Lunch (snacks provided)
14:30 – 15:20	<i>Chair: Frank Wübbeling</i> Joyce McLaughlin (Rensselaer Polytechnic Institute): Stability and statistics for shear stiffness imaging
15:20 – 15:50	Felix Lucka (University College London): Variational methods for dynamic high-resolution photoacoustic tomography
15:50 – 16:10	Coffee Break
16:10 – 16:40	<i>Chair: Matthias Schlottbom</i> Hanne Kekkonen (Helsinki University): Posterior consistency and convergence rates for bayesian inversion

16:40 – 17:05	Oleksandra Panasiuk (RICAM Linz): Regularized ranking by a linear functional strategy
17:05 – 17:30	Galyna Kriukova (RICAM Linz): Regularized ranking: Application in diabetes technology
17:30 – 18:30	Annual Meeting of the GAMM Activity Group MSIP
Day 2 (Tuesday, September 29)	
9:00 - 9:50	<i>Chair: Daniel Tenbrinck</i> Sarang Joshi (University of Utah): Analyzing the changing anatomy
9:50 - 10:20	Hendrik Dirks and Sebastian Suhr: Variational models for simultaneous motion. Estimation and image reconstruction
10:20 – 10:50	Benjamin Berkels (RWTH Aachen): An image registration framework for sliding motion
10:50 - 11:10	Coffee Break
11:10 – 11:40	<i>Chair: Felix Lucka</i> Ville Kolehmainen (University of Eastern Finland): Structural regularization for fMRI
11:40 – 12:10	Federica Sciacchitano (Denmark Technical University): Convex variational approach for restoring blurred images with Cauchy noise
12:10 – 12:40	Andreas Langer (Stuttgart University): Automatic parameter selection for total variation minimization in image restoration
12:40 – 14:00	Lunch (on your own)
14:00 – 14:50	<i>Chair: Martin Benning</i> Nicolas Papadakis (Bordeaux University): Generalized optimal transport models for image processing
14:50 – 15:20	Christoph Brune (Twente University): Joint reconstruction and optimal transport for 4d cell tracking
15:20 – 15:45	Coffee Break
15:45 – 16:15	<i>Chair: Christoph Brune</i> Camille Sutour (Bordeaux University): Adaptive Regularization of the Non-Local means for image and video denoising
16:15 – 16:45	Daniel Tenbrinck (WWU Münster): On the p-Laplacian and infinity-Laplacian on graphs with applications in image and data processing
	Break (Move to castle)
17:30 – 18:30	Distinguished Lecture (lecture hall S 10, Schlossplatz 2, 4 th floor) Stanley Osher (UCLA): Overcoming the Curse of Dimensionality for Hamilton-Jacobi Equations Arising in Control Theory and Elsewhere
19:00	Conference Dinner
Day 3 (Wednesday, September 30)	
9:00 – 9:50	<i>Chair: Michael Möller</i> Marcelo Bertalmio (University Pompeu Fabra): Image processing for enhanced cinematography

9:50 – 10:20	Ronny Bergmann (TU Kaiserslautern): A second-order non-smooth variational model for restoring manifold-valued images
10:20 – 10:50	Leonie Zeune (Twente University): Multiscale segmentation of tumor cells using Bregman iterations
10:50 – 11:20	Coffee Break
11:20 – 11:50	<i>Chair: Martin Burger</i> Philipp Petersen (TU Berlin): Multiscale anisotropic systems on bounded domains
11:50 – 12:20	Rafael Reisenhofer (Bremen University): Complex shearlet-based detection of flame fronts
12:20 – 12:50	Michael Möller (TU Munich): Nonlinear multiscale methods for image and signal analysis
12:50 – 13:00	Closing remarks

Variational Methods for Dynamic Inverse Problems and Imaging

Abstracts

Day 1, Monday, Sept 28

Bernadette Hahn

Department of Applied Mathematics, Saarland University

A time-dependent regularization method for motion compensation

Abstract: The acquisition of tomographic data takes a considerably amount of time. For instance, the x-ray source in computerized tomography has to be rotated around the investigated object. Temporal changes of the object during this time period lead to inconsistent data. Hence, the application of standard reconstruction methods causes motion artifacts in the images which can severely impede the diagnostic analysis. To reduce the artifacts, the regularization method has to take the dynamic behavior of the specimen into account. To obtain an adequate reconstruction, a priori information about the motion is required, which has to be extracted from the measured data. Then, these information are included in specially designed algorithms which compensate for the object's motion within the reconstruction step. In this talk, a suitable regularization method is presented along with numerical results from computerized tomography.

Ozan Öktem

KTH, Stockholm

Shape based regularization

Abstract: We will introduce a framework for regularization of ill-posed linear inverse problems in imaging that can account for shape related a priori information. Shape information is accounted for in the context of classical regularization, but the notion of shape similarity and variability is general enough to be used also in the context of statistical regularization. As a proof of concept, the proposed shape based classical regularization is applied to 2D tomography with very sparse measurements.

Matthias Schlottbom

Institute for Computational and Applied Mathematics, WWU Münster

Diffuse Interface Methods for Inverse Problems: Case Study for an elliptic Cauchy Problem

Abstract: Several inverse problems have to deal with not exactly known geometries. This

makes it desirable to use methods which are robust with respect to perturbed domains. In this talk we discuss a diffuse interface method as a tool for the solution of variational inverse problems. As a particular example we study ECG inversion in some detail. ECG inversion is a linear inverse source problem with boundary measurements governed by an anisotropic diffusion equation. We formulate a regularization strategy using Tikhonov regularization and, using standard source conditions, we prove convergence rates. A special difficulty in the analysis of the regularization method is that not only operator perturbations are introduced by the diffuse interface method, but more important, we have to deal with topologies which depend on a parameter ϵ in the diffuse interface method, i.e. we have to deal with ϵ -dependent forward operators and ϵ -dependent norms. Our results are supported by numerical examples. Joint work with Martin Burger (WWU Münster), Ole L. Elvetun (Norwegian University of Life Sciences)

Martin Benning
Cambridge University

A preconditioned ADMM algorithm for convex optimisation with nonlinear operator constraints.

Abstract: We are presenting a modification of a preconditioned variant of the well-known Alternating Direction Method of Multipliers (ADMM) algorithm that aims at solving convex optimisation problems with nonlinear operator constraints. We further show equivalence to the recently proposed Nonlinear Primal-Dual Hybrid Gradient Method (NL-PDHGM) with extrapolation step on the dual instead of the primal variable. Subsequently, the algorithm is demonstrated to handle nonlinear inverse problem arising in velocity-encoded Magnetic Resonance Imaging (MRI).

Romana Boiger
Alpen-Adria University, Klagenfurt

An online parameter identification method for time-dependent problems

Abstract: Dynamical systems like ordinary differential equations or time-dependent partial differential equations play an important role for modeling instationary processes in science and technology. Such models often contain parameters that cannot be accessed directly and therefore must be determined from measurements, which leads to inverse problems. In case of time-dependent problems the established Tikhonov regularization is not efficient due to the loss of time causality. To avoid this drawback, we use online strategies. In this talk we consider the initial value problem
$$u_t + C(q, u) = f, \quad u(0) = u_0.$$
 The inverse problem we are interested in is to identify q from measurements Gu changing over time, with an observation operator G . The observations in reality are rarely full observations. Hence our aim is to design an online method, that is also applicable for partial observations. For this method we were able to prove conditional convergence. The performance of our approach will be illustrated with some numerical experiments.

Anar Rahimov
Marseille University

On numerical solution on inverse problems for a parabolic equation

Abstract: The study of inverse problems of mathematical physics is conducted in various directions, and in the recent years, the attention of researchers to nonlocal and coefficient-inverse problems has increased. The initial and boundary conditions are specified in nonlocal (integral) form, because it is practically impossible to measure the states of an object (or a process) at individual points or instantly in time. The inverse problems are considered for a parabolic equation with an unknown coefficient depending on only one independent variable (space or time), under initial, final, and boundary conditions. In particular, these classes of problems arise in the study of boundary value problems with nonlocal (integral) boundary conditions. A numerical method is proposed to solve the problems, which is based on the use of the method of lines. The initial problems are reduced to a system of ordinary differential equations with unknown parameters. To solve this system we propose an approach based on the sweep method type. All the necessary computational schemes, formulae, and results of the carried out numerical experiments will be given in the report. The obtained results show the efficiency of the proposed approach.

Joyce R. McLaughlin,
Rensselaer Polytechnic Institute

Stability and Statistics for Shear Stiffness Imaging

Abstract: There are basically two time dependent experiments for shear stiffness imaging. For one, the tissue is excited with a time harmonic oscillation and then sequences of MR data (or less often sequences ultrasound RFQ data sets) are taken and processed to produce a movie of the oscillating tissue within the body. For this experiment we present a set of stability results; some of the results require a single elastic vector movie; other results require a set of elastic vector movies, each movie is the displacement response for a different frequency of oscillation. In the second experiment one pulse or a sequence of pulses are imparted by focusing ultrasound; a wave with a front propagates away from the pulse position. The arrival time of one component of the wave is calculated from the movie created from a sequence of RF/IQ data sets. We establish statistical properties of the noise in the image when using the direct algorithm and show that even though the variance is infinite there are some favorable statistical properties.

Felix Lucka
University College London

Variational methods for dynamic high-resolution photoacoustic tomography

Abstract: The acquisition time of current high-resolution 3D photoacoustic tomography (PAT) devices limits their ability to image dynamic processes in living tissue. In our work, we try to overcome this limitation by combining recent advances in spatio-temporal sub-sampling schemes, variational regularization and convex optimization with the development of tailored data acquisition systems. We first show that images with acceptable spatial resolution can be obtained from suitably sub-sampled PAT data if sparsity-constrained image reconstruction techniques such as total variation (TV)

regularization enhanced by Bregman iterations are used. A further increase of the dynamic frame rate can be achieved by exploiting the temporal redundancy of the data through the use of sparsity-constrained dynamic models. While simulated data from numerical phantoms will be used to illustrate the potential of the developed methods, we will also discuss the results of their application to different measured data sets. Furthermore, we will outline how to combine GPU computing and state-of-the-art optimization approaches to cope with the immense computational challenges imposed by 4D PAT. Joint work with Marta Betcke, Simon Arridge, Ben Cox, Nam Huynh, Edward Zhang and Paul Beard.

Hanne Kekkonen
Helsinki University, Finland

Posterior Consistency and Convergence Rates for Bayesian Inversion

Abstract: Let us consider an indirect noisy measurement M of a physical quantity U $M = AU + d \cdot E$ where the measurement M , noise E and the unknown U are treated as random variables and d models the noise amplitude. We are interested to know what happens to the approximate solution of above when $d \rightarrow 0$. The analysis of small noise limit, also known as the theory of posterior consistency, has attracted a lot of interest in the last decade, however, much remains to be done. Developing a comprehensive theory is important since posterior consistency justifies the use of the Bayesian approach the same way as convergence results do the use of regularisation techniques.

Oleksandra Panasiuk
RICAM Linz

Regularized ranking by a linear functional strategy

Abstract: In this talk we explore the possibility to employ regularization schemes for performing ranking tasks. We discuss a rather general approach to the regularization in reproducing kernel Hilbert space (RKHS) with the idea to mimic the best linear combination of the calculated regularized rankers corresponding to different values of the regularization parameter. The main result is the choice of the coefficients of the above mentioned linear combination that is based on the so-called linear functional strategy. We also discuss several applications where the proposed ranking algorithm can be effectively used.

Galyna Kriukova
RICAM Linz

Regularized ranking: Application in diabetes technology

Abstract: The ranking algorithms have become a standing topic in learning theory recently because of their importance for the development of new recommender systems. In this talk, we illustrate how the regularized ranking algorithm can be applied for diabetes treatment. We also discuss the application of aggregation by linear functional strategy and collaborative filtration to predict the occurrence of nocturnal hypoglycemia for diabetes patients.

Day 2, Tuesday Sept 29

Sarang Joshi
University of Utah

Analyzing the Changing Anatomy

Abstract: In this talk I will present computational and analytical tools we have been developing at University of Utah for the analysis of anatomical image ensembles that are designed to capture changes in anatomy. The fundamental analytical framework we have been using is that of regression analysis where the dependent variable is the anatomical configuration while the independent variable is application domain specific. I will exemplify the application of this general methodology to various medical imaging applications ranging from the analysis of Internal Organ Motion as imaged in 4D respiratory correlated CT imaging spanning few minutes to that of the study changes in brain anatomy associated with normal aging and neurodegenerative diseases such as Alzheimer's spanning decades.

Hendrik Dirks and Sebastian Suhr
WWU Münster

Variational Models for Simultaneous Motion Estimation and Image Reconstruction

Abstract: In many fields of image processing, especially in medicine, the recorded data is usually distorted by noise but at the same time coupled by some internal dynamics. Both problems are directly connected and therefore offer several ways of simultaneous estimation. In this talk we describe joint models from the perspective of optical flow on the one hand and image registration on the other. Besides some introductory modeling we present an overview of analytical results and highlight the potential of the methods towards data with poor signal-to-noise ratio. Moreover we shortly outline possible numerical implementations.

Benjamin Berkels
RWTH Aachen

An image registration framework for sliding motion

Abstract: We present a novel variational framework for image registration with explicit modeling of sliding motion, as it occurs, e.g., in the medical context at organ boundaries. The key of our method is a piecewise smooth deformation model that allows for discontinuities at the sliding interfaces while keeping the sliding domain in contact with its surrounding. The presented approach is generic and can be used with a large class of both image similarity measures and regularizers for the deformation. A useful byproduct of the proposed method is an automatic propagation of a given segmentation from one image to the other. Existence of minimizers of the underlying variational formulation can be shown under rather mild assumptions. The numerical minimization is based on a splitting approach with alternating derivative based Gauss-Newton and fast first order convex optimization. This is joint work with S. Heldmann, T. Polzin and A. Derksen.

Ville Kohlehmainen
University of Eastern Finland

Structural regularization for fMRI

Abstract: In this talk, we present construction of structural regularization for functional MRI with sparsely sampled data. The anatomical prior information for the construction of the regularization functional is extracted from a densely sampled MRI frame which is acquired as part of the fMRI measurement protocol. The approach is evaluated using simulated measurement data and experimental data taken from a Wistar rat specimen using a 9.4T small animal MRI scanner.

Federica Sciacchitano
Denmark Technical University

Convex variational approach for restoring blurred images with Cauchy noise

Abstract: Image restoration is a challenging task in applied mathematics, since it generally requires to solve an ill-posed inverse problem. During the years, many variational models have been introduced to handle the restoration problem with Gaussian noise, multiplicative noise and impulse noise. In this talk, we propose a variational method for deblurring and denoising degraded images with a very impulsive noise, the so-called Cauchy noise. In order to obtain a strictly convex model, which guarantees the uniqueness of the solution, a quadratic penalty technique is used. Numerical experiments show that the proposed approach outperforms the other well known methods.

Andreas Langer
Stuttgart University

Automatic Parameter Selection for Total Variation Minimization in Image Restoration

Abstract: A good approximation of the original image from an observed image may be obtained by minimizing a functional that consists of a data-fidelity term, a regularization term, and a parameter, which balances data-fidelity and regularization. The proper choice of the parameter is delicate. In fact, large weights not only remove noise but also details in images, while small weights retain noise in homogeneous regions. Hence a good reconstruction may be obtained by choosing the parameter such that a good compromise of the aforementioned effects are made. Based on the discrepancy principle we present an algorithm which automatically computes a good scalar regularization parameter. However, since images consist of multiple objects of different scales, it is expected that a spatially varying weight would give better reconstructions than a scalar parameter. In this vein we adapt our proposed algorithm for computing a distributed weight. We study the convergence behaviour of the proposed algorithms and present numerical experiments for Gaussian noise removal as well as for impulsive noise removal.

Nicolas Papadakis
Bordeaux University

Generalized optimal transport models for image processing

Abstract: Optimal Transport is a well developed mathematical theory that defines a family of metrics between probability distributions. The ability to compute optimal displacements between densities through the corresponding transport map makes this theory progressively mainstream in several applicative fields.

For Image Processing applications, the transport map can for instance be used to compute geodesics between images or to transfer characteristics of one image to another.

In this context, it is of major interest to preserve the nature of the objects contained in the images, in order to synthesize images that are visually plausible.

This can be done by imposing some physical regularity to the transport map to estimate.

In this talk, some regularized optimal transport models will thus be presented, both in continuous and discrete formulations of Optimal Transport.

The interest of these generalized models will be illustrated with applications to image interpolation and image transfer.

Christoph Brune

Twente University

Joint reconstruction and optimal transport for 4d cell tracking

Abstract: In dynamic biomedical imaging mathematical inversion and tracking methods play a fundamental role. Particularly for tomography and live microscopy innovative spatio-temporal imaging models are of strongly growing interest. The aim of this talk is to highlight recent modeling, analysis and computations of joint density reconstruction and flow quantification for 4d image sequences. With the success of TV and TGV regularization for static imaging we focus on properties of spatio-temporal TV priors and compressive sensing for optimal transport.

Camille Sutour

Bordeaux University

Adaptive regularization of the Non-Local means for image and video denoising

Abstract: We focus on adaptive image and video denoising. The non-local means (NL-means) perform denoising by exploiting the natural redundancy of patterns inside an image; they perform a weighted average of pixels whose neighborhoods (patches) are close to each other. This reduces significantly the noise while preserving most of the image content. While it performs well on flat areas and textures, it suffers from two opposite drawbacks: it might over-smooth low-contrasted areas or leave a residual noise around edges and singular structures. Denoising can also be performed by total variation minimization – the ROF model – which leads to restore regular images, but it is prone to over-smooth textures, staircasing effects, and contrast losses. We introduce in this paper a variational approach that corrects the over-smoothing and reduces the residual noise of the NL-means by adaptively regularizing non-local methods with the total variation. The proposed regularized NL-means algorithm combines these methods and reduces both of their respective defaults by minimizing an adaptive total variation with a non-local data fidelity term. Besides, this model adapts to different noise statistics and a fast solution can be obtained in the general case of the exponential family. We develop this model for image denoising and we adapt it to video denoising with 3D patches.

Daniel Tenbrinck
WWU Münster

On the p -Laplacian and Infinity-Laplacian on graphs with applications in image and data processing

Abstract: The variational p -Laplacian and ∞ -Laplacian have been of particular interest in applied sciences ever since as they allow to model many phenomena in physics or biology. Some related variants known as nonlocal Laplacian and game- p -Laplacian have gotten more and more in the focus of research as they play an important role in, e.g., continuum mechanics, population dynamics, and game theory.

Recently, graph-based methods have emerged as a promising tool for many applications in machine learning and data processing. One key feature of these methods is the possibility to exploit nonlocal relationships in the data rather than using only local neighborhoods. The recent trend in the literature is to translate well-studied variational problems and PDEs to the graph setting and overcome hereby drawbacks of classical approaches. Especially the translation of the p -Laplacian and ∞ -Laplacian to graphs is of particular interest for many applications in machine learning and image processing.

In this talk we introduce a new family of partial difference operators on graphs and study equations involving these operators. This family covers local and nonlocal variational p -Laplacian and ∞ -Laplacian as well as gradient operators used in morphology. We analyze a corresponding parabolic equation involving these operators which enables us to interpolate adaptively between p -Laplacian diffusion-based filtering and morphological filtering, i.e., erosion and dilation. In the case $p = \infty$ we investigate a connection to a stochastic game known as Tug-of-War and give a possibility to introduce nonlocality into this setting. Finally, we demonstrate the adaptability of the proposed formulation for different tasks in image and point cloud processing, such as filtering, segmentation, clustering, and inpainting.

Stanley J. Osher
University of California Los Angeles

Overcoming the Curse of Dimensionality for Hamilton-Jacobi Equations Arising in Control Theory and Elsewhere

Abstract: Using techniques related to compressive sensing reconstruction, we solve an important class of initial value problems for Hamilton-Jacobi. We can solve problems in high dimensions and each evaluation of the solution can be done in between $10^{(-6)}$ and $10^{(-8)}$ seconds on a laptop. Additionally, we can locate the closest point to the union of a collection on convex compact sets in high dimensions equally quickly. (joint work with Jerome Darbon)

Day 3, Wednesday Sept 30

Marcelo Bertalmio
University Pompeu Fabra

Image processing for enhanced cinematography

Abstract: Despite that nowadays most cameras, even low-end consumer models, record video that when displayed on any common screen looks good and appears faithful to the original scene, on closer inspection the latter is never the case: there are details, visible at the scene, that are lost in the picture; conversely, there are details that we may only see in the picture, but were not noticeable at the scene; and colors never quite match, they don't have the same shade, or saturation or brightness as in the original location. It is due to camera and display limitations that the shooting location and the images on the screen are perceived very differently. Adding extra lights on the location helps to improve image contrast, but this isn't usually enough, and it's also a very time consuming and expensive process. Trying to overcome the difference in display capabilities, professional movies are color-corrected shot by shot by skilled technicians, to generate outputs for different targets (e.g. digital cinema projectors and standard definition TV), but the results leave much room for improvement.

This talk will present an overview of our ongoing work in image processing algorithms for cinema aiming to allow people watching a movie on a screen to see the same details and colors as people at the shooting location can.

Ronny Bergmann
TU Kaiserslautern

A second-order non-smooth variational model for restoring manifold-valued images

Abstract: In many real world situations, measured data is noisy and nonlinear, i.e., the data is given as values in a certain manifold. Examples are InSAR images and the hue channel of HSV, where the entries are phase-valued, directions in \mathbb{R}^n , which are data given on S^{n-1} , and diffusion tensor magnetic resonance imaging, where the obtained data items are symmetric positive definite matrices. In this talk we extend the recently introduced total variation model on manifolds by a second order TV type model. We first introduce second order differences on manifolds in a sound way using the induced metric on Riemannian manifolds. By avoiding a definition involving tangent bundles, this definition allows for a minimization employing the inexact cyclic proximal point algorithm, where the proximal maps can be computed using Jacobian fields. The algorithm is then applied to several examples on the aforementioned manifolds to illustrate the efficiency of the algorithm.

Leonie Zeune
Twente University

Multiscale segmentation using spectral TV analysis

Abstract: In cancer research one of the most important challenges is the early detection

and guided therapy of patients. One approach is the accurate quantification of circulating tumor cells (CTC) in blood samples. To establish this as a clinical routine there is a strong need for an automatic and reliable segmentation method for various cell types. The main goal of this talk is to present a robust multiscale framework for TV-based image segmentation. We compare the forward scale-space to the inverse scale-space using Bregman iteration. A particular focus is the automatic detection of multiple scales using spectral decomposition recently introduced for TV denoising. The effectiveness of the proposed method is shown by results for experimental multi-channel cell data.

Philipp Petersen
TU Berlin

Multiscale anisotropic systems on bounded domains

Abstract: At the core of many inverse problems lies a physical process that is often governed by a partial differential equation. In order to solve these PDEs numerically some effort needs to be made to find the best discretization. During the last decades a trend for the solution of PDEs emerged, that focuses on employing systems from applied harmonic analysis for the adaptive solution of these equations. Most notably wavelet systems have been used and lead for instance to provably optimal solvers for elliptic PDEs. Inspired by this success story also other systems with various advantages in different directions should be employed in various discretization problems. For instance, ridgelets were recently successfully used in the discretization of linear transport equations. Another famous system is that of shearlets, which admits optimal representations of functions that have singularities along smooth curves. However, in order to apply these methods in adaptive discretization algorithms it is necessary to have a system on a bounded domain, which yields a frame, is able to incorporate boundary conditions, and characterizes Sobolev spaces. Although there have been first approaches to construct shearlet systems for the solution of PDEs on bounded domains they fail to satisfy all the desiderata above. In this talk we will introduce a novel shearlet system that meets all the requirements mentioned above and admits optimal approximation rates for functions with discontinuities along curves.

Rafael Reisenhofer
Bremen University

Complex shearlet-based detection of flame fronts

Abstract: Results of an ongoing collaboration with the Division of Engineering Thermodynamics of the University of Bremen are presented, in which complex shearlet-based edge and line measures are used to determine the location and curvature of flame fronts in high speed recordings of flames. The complex shearlet transform is a complex-valued generalization of the shearlet transform [1] in which anisotropically (i.e. in a directionally-dependent fashion) scaled and sheared wavelet-like basis functions provide sparse approximations of signals in two and higher dimensions. While shearlet-based transforms were in fact shown to provide optimally sparse approximations [2] of elements within a certain class of natural images, complex-valued shearlet transforms are conceptually closer to the classical Fourier transform in the sense that a complex shearlet $\psi^c = \psi + i\mathcal{H}\psi$ consists of a real-valued shearlet ψ and its Hilbert transform $\mathcal{H}\psi$, where ψ is typically chosen to be symmetric, forcing $\mathcal{H}\psi$ to be odd-symmetric. A structure, which resembles the complex exponential $e^{ix} = \cos(x) + i\sin(x)$, where the real part consists of the even-symmetric cosine, the

imaginary part consists of the odd-symmetric sine and both sine and cosine just differ by a 90 degree phase shift. For edge detection, we exploit the fact that, given the right normalization, both the real and the imaginary part of a complex-valued shearlet transform exhibit a specific scale-independent behavior at the location of an edge. This observation leads to a computationally simple edge measure which is by construction contrast invariant, stable in the presence of noise and capable of approximating tangent orientations. Furthermore, by reversing the roles of the real and the imaginary parts of a complex-valued shearlet transform in the definition of said measure, lines can be detected instead of edges. References: [1] Guo, Kutyniok and Labate: Sparse Multidimensional Representations using Anisotropic Dilation and Shear Operators, 2005 [2] Guo and Labate: Optimally Sparse Multidimensional Representation using Shearlets, 2007

Michael Möller
TU Munich

Nonlinear Multiscale Methods for Image and Signal Analysis

Abstract: Classical strategies for image and signal analysis apply an orthonormal linear transform, filter the resulting coefficients, and apply the inverse transform. If the filtering itself is also linear, one typically interprets the whole process with the help of the singular value decomposition of the linear operator. This talk discusses the possibility to introduce similar concepts for nonlinear transformations arising from the solution of variational, scale space, or inverse scale space methods. We introduce the notion of generalized frequency and wavelength representations as well as filterings of the corresponding coefficients. First numerical results demonstrate interesting properties of the resulting image decomposition.