

4th Applied Mathematics Symposium Münster

# Analytical and numerical aspects of wave propagation in complex structures

## Speakers

Assyr Abdulle (EPFL)  
Antoine Benoit (Université Littoral)  
Andreas Buhr (WWU Münster)  
Tomas Dohnal (TU Dortmund)  
Jan-Philip Freese (KIT)  
Dietmar Gallistl (KIT)  
Patrick Joly (ENSTA ParisTech)  
Jens Markus Melenk (TU Vienna)  
Olof Runborg (KTH Stockholm)  
Maik Urban (TU Dortmund)  
Barbara Verfürth (WWU Münster)

For further information please visit

<http://www.uni-muenster.de/AMM/wave2018/>

## Organizers

Patrick Henning, Agnes Lamacz, Mario Ohlberger, Ben Schweizer, Maik Urban, Barbara Verfürth



## General information:

Welcome to the 4th Applied Mathematics Symposium in Münster:

***Analytical and numerical aspects of wave propagation in complex structures***

In the following, we will give you some information on the activities and the schedule of the workshop.

We have a lunch break on Thursday and Friday in the main hall of the Seminar building (Map, No 5)

The conference dinner will take place on Thursday, March 15, 2018 at 6.30 p.m in the restaurant Ideal in Münster.

You will find a map how to get there.

Wifi access is available. If you need Wifi Access, please ask for a password.

Alternatively, if you are part of the eduroam community, you may connect to the network "eduroam" as usual.

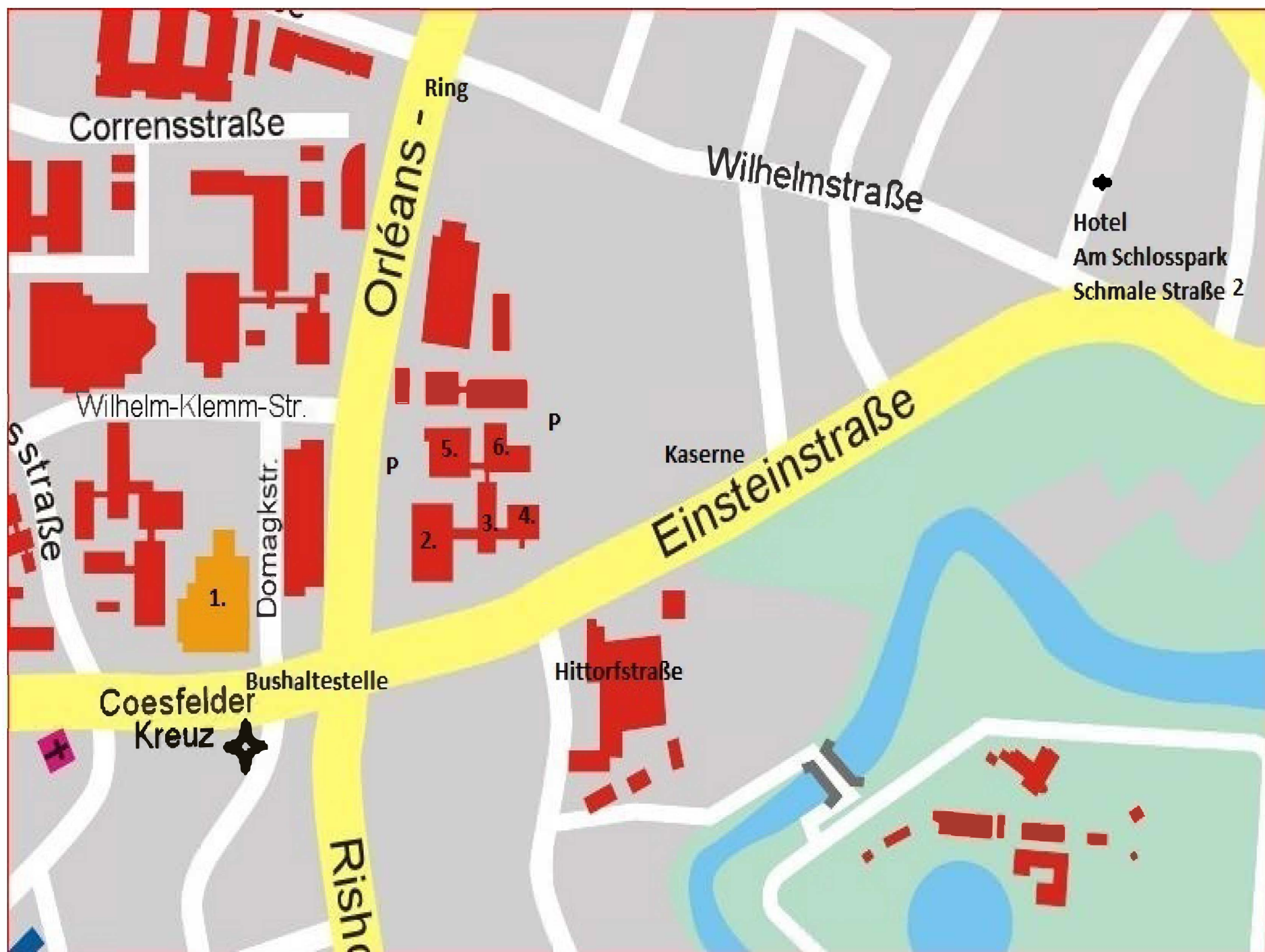
Information on the invited talks of the workshop are provided in the attached book of abstracts. All talks will take place at SRZ 5 (Map, No 5)

### **Information for speakers:**

You may use your own notebook and presenter for your presentation, or the ones provided in the lecture room.

Please make sure that everything works before your actual talk (e.g. in one of the coffee/lunch breaks before your session)

In case of questions, we are happy to assist you. You find contact persons at the registration desk and during the breaks.



1. Mensa

2. Hörsaalgebäude / Lecture Hall

Einsteinstr. 64

3. Hochhaus Mathematik/Informatik / highrise Mathematics/  
Informatics

Einsteinstr. 62

4. ZIV

5. Seminargebäude / seminar building

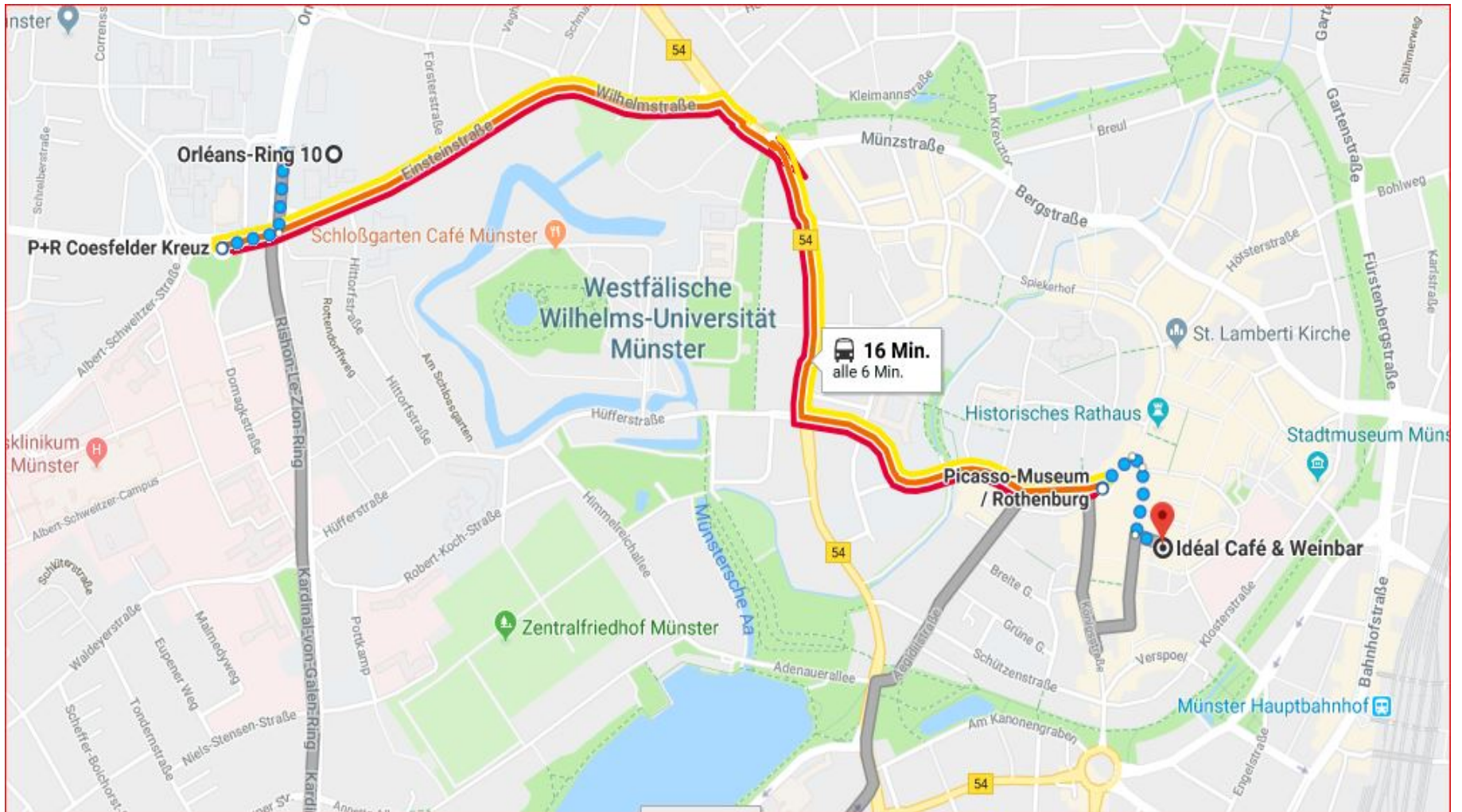
Orléansring 12

6. Angewandte Mathematik / Applied Mathematics

Orléansring 10

P- Parkplatz / parking

\* Bushaltestelle Coesfelder Kreuz / bus stop Coesfelder Kreuz



The conference Dinner will take place at the Restaurant IDEAL which is located at the Beguinenstraße 12 in Münster.

You can use Bus No. 11 from Coesfelder Kreuz direction Tannenhof and get off at Picasso Museum. (University)

You can use Bus No. 9 from Steinfurter Straße direction Franz-Marc-Weg / Prinzipalmarkt and get off at Picasso Museum (Hotel Am Schlosspark)

You can use Bus No. 14 from Jungeboldtplatz direction Gallenkamp and get off at Picasso Museum (Hotel Jellentrup)

The IDEAL is in walking distance to the Picasso Museum

The conference dinner starts at 6.30 p.m.

Thursday, March 15 Time	Speaker / Program	Title
10:00-10:15	Registration and Opening Main Hall and Room 5, Seminar building	
10:15-11:00	Antoine Benoit (Université Littoral)	Long time homogenization for the classical wave equation
11:00-11:45	Assyr Abdulle (EPFL)	Multiscale methods for advection-diffusion problems with large compressible flows
11:45-12:15	Jan-Philip Freese (KIT)	Towards numerical homogenization of the Maxwell- Debye system: Semidiscrete error analysis
12:15-13:30	Lunch break	
13:30-14:15	Maik Urban (TU Dortmund)	Homogenization of time- harmonic Maxwell's equations in general periodic microstructures
14:15-15:00	Barbara Verfürth (University of Münster)	Numerical multiscale methods for Maxwell's equations in periodic media
15:00-15:30	Coffee break	
15:30-16:15	Patrick Joly (ENSTA ParisTech)	Topographic open waveguides: mathematical and computational aspects
16:15-17:00	Tomas Dohnal (TU Dortmund)	Waves in Periodic Structures: 1) A Radiation Boundary Condition Based on Bloch Waves 2) Asymptotics of Wavepackets in Nonlinear Media
17:00-17:45	Matthias Schlottbom (University Twente)	Perfectly matched layer for the radiative transfer equation
18:30-	Conference Dinner	

Friday, March 16 Time	Speaker / Program	Title
09:30-10:15	Olof Runborg (KTH Stockholm)	Multiscale methods for wave propagation problems
10:15-11:00	Andreas Buhr (University of Münster)	Simulation of electromagnetic fields in highly complex printed circuit boards using localized model order reduction
11:00-11:30	Coffee break	
11:30-12:15	Jens Markus Melenk (TU Vienna)	Wavenumber-explicit hp-FEM for Maxwell's equations
12:15-13:00	Dietmar Gallistl (KIT)	On the algorithmic use of commuting quasi-interpolation in the numerical homogenization of H(curl) problems
13:00-14:00	Lunch break  End of Workshop	

# Multiscale methods for advection-diffusion problems with large compressible flows

Assyr Abdulle

École Polytechnique Fédérale de Lausanne

Numerical homogenization methods for the numerical approximation of parabolic advection-diffusion problems with rapidly varying coefficients, large Péclet number and compressible flows are discussed. The multiscale scheme is based on the coupling of macro and micro finite element methods and relies on discontinuous Galerkin discretizations. Sharp a priori error estimates which are robust in the advection dominated regime are derived. We will illustrate the applicability of the method at various numerical examples.

## References

- [1] A.Abdulle, M.Huber, *Numerical homogenization method for parabolic advection-diffusion multiscale problems with large compressible flows*, Numer. Math. 136, 2017.
- [2] A. Abdulle, *Numerical homogenization methods for parabolic monotone problems*. In: *Building bridges: connections and challenges in modern approaches to numerical partial differential equations*, 1–38, Lect. Notes Comput. Sci. Eng., 114, Springer, [Cham], 2016.
- [3] A.Abdulle, M.Huber, *Discontinuous Galerkin finite element heterogeneous multiscale method for advection-diffusion problems with multiple scales*, Numer. Math. 126, 2014.

# Long time homogenization for the classical wave equation

Antoine Benoit

Université Littoral Côte d'Opal

In this talk we are interested in the classical wave equation :

$$\begin{cases} \partial_{tt}^2 u^\varepsilon - \nabla \cdot a\left(\frac{x}{\varepsilon}\right) \nabla u^\varepsilon = 0, & t \in [0, T], x \in \mathbb{R}^d, \\ u^\varepsilon|_{t=0} = u_0, \\ (\partial_t u^\varepsilon)|_{t=0} = 0, \end{cases}$$

where the field  $a$  encodes the response of the media for the wave propagation. The aim in homogenization is then to describe a partial differential equation which is solved by  $u$  the limit of  $u^\varepsilon$  when  $\varepsilon \downarrow 0$ .

When  $a$  is periodic, the classical homogenization result states that for time scales of order  $T > 0$ ,  $u$  solves an homogeneous wave equation for a homogenized field  $a_{hom}$ . This classical result has been improved to time scales of order  $\varepsilon^{-2}T$  by [Dohnal-Lamacz-Schweizer '14-'15]. More precisely, the authors show that for time scales of order  $\varepsilon^{-2}T$ , some dispersive effects become significant, so that a new homogenized equation including these effects has to be introduced.

In this talk, we will explain how we can extend this result to higher order time scales (typically  $\varepsilon^{-\ell}T$ ,  $\ell \in \mathbb{N}$ ) and to more general coefficients fields (typically to stochastic ones). Our method is a generalization of the so-called Bloch waves decomposition already used in [Dohnal-Lamacz-Schweizer '14-'15] for periodic media. This method referred as Taylor-Bloch method consists in the construction of approximate eigenmodes/eigenvalues for the magnetic operator  $-(\nabla + ik) \cdot a(\nabla + ik)$  and is linked, in the homogenization point of view, to a new set of extended correctors.



## Waves in Periodic Structures:

- 1) A Radiation Boundary Condition Based on Bloch Waves
- 2) Asymptotics of Wavepackets in Nonlinear Media

Tomas Dohnal

TU Dortmund

In the first part of the talk we present a novel radiation boundary condition for scattering problems in two dimensional infinite waveguides, where the medium is periodic far away from the scatterer/interface. The scheme attaches radiation boxes to the truncated domain and expands the solution in outgoing Bloch waves in these boxes. We prove existence and stability of the resulting infinite dimensional model and provide some numerical tests.

In the second part of the talk we study wavepackets of small amplitude and large spectral concentration in cubically nonlinear wave problems in periodic media. In particular, the Gross-Pitaevskii equation and a nonlinear wave equation are considered. In the case of a single carrier Bloch wave we justify the nonlinear Schrödinger equation as an effective asymptotic model for the slowly varying envelope in  $d \in \mathbb{N}$  spatial dimensions. In one and two dimensions we also consider wavepackets with multiple carriers leading to a system of first order coupled mode equations. One of the aims of the asymptotics is the prediction of close to solitary waves in these nonlinear periodic problems.

# Simulation of Electromagnetic Fields in Highly Complex Printed Circuit Boards using Localized Model Order Reduction

Andreas Buhr

University of Münster

(joint work with Mario Ohlberger)

The design of modern high speed electronics with frequencies in the range of multiple gigahertz requires simulation of the electromagnetic fields involved. Approximation of the solution of the corresponding time harmonic Maxwell equation in structures like IC packages or printed circuit boards using the finite element method is challenging, as these structures are extremely complex while having no periodicity and no scale separation which can be exploited. We numerically analyze the applicability of localized model order reduction schemes to this problem. On the example of an Olimex OLinuXino A64 mini PC [1] (Raspberry Pi like), we define local transfer operators [2, 4] whose left singular vectors span optimal local approximation spaces and analyze the decay of their singular values. The required basis sizes and thus the reduction potential is discussed. Furthermore, we report CPU times for the generation of local approximation spaces using methods from randomized linear algebra [3].

## References

- [1] Olimex OLinuXino A64, <https://www.olimex.com/Products/OLinuXino/A64/>, accessed: 2018-02-15.
- [2] Ivo Babuska and Robert Lipton, *Optimal local approximation spaces for generalized finite element methods with application to multiscale problems*, *Multiscale Modeling & Simulation*, 9(1):373–406, 2011.
- [3] Andreas Buhr and Kathrin Smetana, *Randomized local model order reduction*, Technical Report arXiv:1706.09179, June 2017.
- [4] Kathrin Smetana and Anthony T. Patera, *Optimal local approximation spaces for component-based static condensation procedures*, *SIAM J. Sci. Comput.*, 38(5):A3318–A3356, 2016.

# Towards numerical homogenization of the Maxwell-Debye system: Semidiscrete error analysis

Jan-Philip Freese

Karlsruhe Institute of Technology

In this talk we investigate time dependent Maxwell's equations coupled with the Debye model for orientation polarization in a medium with highly oscillatory parameters. The goal is to characterize the macroscopic behavior of the solution to the resulting integro-differential system. We use analytical homogenization results to derive the effective Maxwell system with the corresponding cell problems, which in this case lead to time dependent cell correctors and thus to further challenges for a numerical approach. The Finite Element Heterogeneous Multiscale Method (FE-HMM) is applied to solve the homogenized Maxwell system and we give first insights into the semidiscrete error analysis.

# On the algorithmic use of commuting quasi-interpolation in the numerical homogenization of $H(\text{curl})$ problems

Dietmar Gallistl

Karlsruhe Institute of Technology

(joint work with P. Henning and B. Verfürth)

If an elliptic differential operator associated with an  $H(\text{curl})$  problem involves rough (rapidly varying) coefficients, then solutions to the corresponding  $H(\text{curl})$  problem admit typically very low regularity, which leads to arbitrarily bad convergence rates of conventional numerical schemes. This contribution shows how the missing regularity can be compensated through a corrector operator. Such operators were constructed by Målqvist and Peterseim (2014) for diffusion problems by combining ideas from the Variational Multiscale Method with quasi-interpolation operators. In this contribution, the local bounded cochain projection of Falk and Winther (2014) is proposed as part of the algorithm. The resulting decomposition gives an optimal order approximation in  $H(\text{curl})$ , independent of the regularity of the exact solution. Furthermore, the corrector operator can be localized to patches of macro elements, which allows an efficient computation. The coarse part of the numerical solution is still a meaningful approximation in the dual of  $H(\text{div})$ .

# Topographic open periodic waveguides: mathematical and computational aspects

Patrick Joly

ENSTA ParisTech & INRIA

(joint work with B. Delourme, S. Fliss and E. Vasilevskaya)

We shall consider, in the 2D setting, the propagation of electromagnetic waves in a homogeneous dielectric medium with a biperiodic array of perfectly conducting rectangular inclusions. We shall show that by a lineic (and purely geometric) perturbation of the medium, it is possible to create a waveguide. The proof is based on a perturbation approach and asymptotic analysis. We shall also present two numerical methods for computing approximately the guided modes and their dispersion relation.

# Wavenumber-explicit hp-FEM for Maxwell's equations

Jens Markus Melenk

TU Vienna

(joint work with Stefan Sauter (Zurich))

We consider the time-harmonic Maxwell equation in homogeneous media at large wavenumbers  $k$  discretized by high order edge elements. For a model problem, we show that quasi-optimality is achieved if (a) the approximation order  $p$  is selected as  $p = O(\log k)$  in conjunction with (b) a mesh size  $h$  such that  $kh/p$  is small. As in the related case of the Helmholtz equation, the analysis relies on a  $k$ -explicit regularity theory for Maxwell's equations that decomposes solutions into two components: the first component is an analytic, but highly oscillatory function while the second one has finite regularity but features wavenumber-independent bounds. An issue particular to Maxwell's equations that is not present in the case of the Helmholtz equations is the problem of approximating discrete divergence-free functions by divergence-free ones. In the present  $hp$ -version context, this is possible at the optimal rate due to a suitable projection-based interpolation operator that enjoys the commuting diagram property.

# Multiscale methods for wave propagation problems

Olof Runborg  
KTH Stockholm

Wave propagation problems with rapidly varying coefficients are computationally costly to solve by traditional techniques because the smallest scales must be represented over a domain determined by the largest scales of the problem. We consider numerical methods for such wave propagation in the framework of the heterogeneous multiscale method and finite differences. These methods couple simulations on macro- and microscales for problems with rapidly oscillating coefficients. The complexity is significantly lower than that of traditional techniques with a computational cost that is essentially independent of the microscale. In this talk we show analysis of how the method works in the long time case, where the macroscale solution exhibit dispersive behavior.

# Perfectly matched layers for the radiative transfer equation

Matthias Schlottbom

University of Twente

(joint work with H. Egger, TU Darmstadt)

The radiative transfer equation (RTE) is a hyperbolic integro-partial differential equation that describes the dynamics of a single-particle probability distribution in location-velocity phase space. The dynamics is governed by streaming, damping and scattering. In scattering dominated problems, velocity semi-discretizations based on truncated spherical harmonics expansions are widely used. A major difficulty in such approaches is the derivation of boundary conditions for the coefficient functions. Using variational formulations and subsequent Galerkin projection, boundary conditions can be constructed systematically. Such formulations couple, however, all coefficient functions, thereby increasing the computational costs tremendously.

In this talk, we present a two-step strategy for obtaining a numerical scheme that overcomes these problems. In a first step, we introduce an absorbing layer, which leads to an exponential damping of the solution to the RTE. In a second step, we introduce specifically tailored artificial reflection boundary conditions, which are shown to be a minor perturbation. Using this formulations, we derive a mixed variational formulation for the RTE that removes the coupling of the spherical harmonics expansion coefficient functions, i.e., yielding a numerical scheme with optimal computational complexity. Our error estimates are confirmed by numerical experiments.



# Homogenization of the time-harmonic Maxwell equations in general periodic microstructures

Maik Urban

TU Dortmund

Inspired by new experimental observations, homogenization of the time-harmonic Maxwell equations in periodic meta-materials has been an active field of research for the last 20 years. We contribute with a study of periodic meta-materials with period  $\eta > 0$  consisting of perfectly conducting microstructures and void space. Most of the known results treat a microstructure with a particular topology. By contrast, we discuss the homogenization of Maxwell's equations for a general class of microstructures. The topological characteristics of the medium determine the structure of the macroscopic equations as well as the transmission properties of the effective medium.

# Numerical multiscale methods for Maxwell's equations in periodic media

Barbara Verfürth

University of Münster

The propagation of electromagnetic fields in periodic materials is considered with growing interest as these media can show unusual behavior, such as frequency band gaps and even negative refraction. To produce such effects, the materials possess some (periodic) sub-wavelength fine-scale structures with a high contrast between at least two composites. The simulation of such problems is quite challenging due to the general wave nature of the problem and the additional fine-scale oscillations from the material inhomogeneities. At the example of the Helmholtz and the time-harmonic Maxwell's equations, we show that the high contrast leads to an effective permeability with negative real part at certain frequencies in the homogenization limit. A (fine-scale) corrector is needed for a good approximation in  $L^2$  and it can be efficiently computed as the solution of local cell problems. A priori error analysis and numerical examples for the corresponding numerical multiscale method are presented.

## List of Participant:

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