Probability and dynamics

Seminar, SoSe 2024

This seminar will treat some ideas and results at the intersection of probability and dynamics, organized around the themes of entropy, percolation, and the geometry of groups. Basic references are the books [6] and [7]. The following are proposed topics:

(1) basic ergodic theory: p.m.p. actions, ergodicity, mixing, Bernoulli actions (§2.1-2.3 of [6])
(2) amenability, soficity, and property (T) (including Glasner–Weiss dynamical-probabilistic characterization [4]) for discrete groups (§4.1, 5.1, 10.2 of [6]; [1])
(3) measure entropy for p.m.p. actions of amenable and sofic groups (§9.1-9.5, 10.1, 10.3 of [6])
(4) sofic generator theorem, computation of sofic entropy for Bernoulli actions via second moment method (§10.4, 10.5 of [6])
(5) Bernoulli bond percolation: first and second moment methods, critical probability, example of trees (§5.2, 5.3 of [7])
(6) number of infinite clusters in Bernoulli percolation, amenability implies at most one infinite cluster (§7.3 of [7])
(7) nonamenability and $p_\mathrm{c} < p_\mathrm{u}$ ([8] and §7.7 of [7])
(8) characterization of amenability in terms of invariant percolation (Theorem 1.1 of [2]; makes use of the mass-transport principle: §8.1 in [7])
(9) Gaboriau–Lyons theorem: nonamenable groups measurably contain free groups [3]
(10) Hutchcroft–Pete theorem: property (T) groups have cost one [5] (makes use of the mass-transport principle: §8.1 of [7])

References

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