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Winter Semester 2020/21
01/25/2021

Probability Theory on Trees and Networks

Exercise Sheet 11

Submission is due on 02/01/2021 9 p.m.

Please send the solutions to yannic.broeker@uni-muenster.de as a pdf-file.

Exercise 1 (5 points)

As in the lecture we define

$$u(\psi, x) = \mathbb{E}_{(\psi,x)}\left[1\left\{\lim_{n\to\infty} \psi_n(0) = 1\right\}\right].$$

Prove that this u is harmonic.

Exercise 2 (5 points)

Let G be a transient, infinite graph. Give a clean proof of the following implication:

the SRW on G is transient $\quad\Rightarrow\quad$ the lamplighter graph $G^{\odot}$ has not the Liouville property.

Exercise 3 (5 points)

Again, let G be a transient, infinite graph. Now prove the other direction:

the lamplighter graph $G^{\odot}$ has not the Liouville property $\quad\Rightarrow\quad$ the SRW on G is transient.

Hint: Use Exercise 4 from Exercise Sheet 10.

Exercise 4 (5 points)

Let $(M_1, \mathcal{F}_1, Q_1)$ and $(M_2, \mathcal{F}_2, Q_2)$ be two probability spaces, and suppose that $\Phi : (M_1, \mathcal{F}_1) \to (M_2, \mathcal{F}_2)$ is measurable with $Q_2 = Q_1 \circ \Phi^{-1}$. Show that the following are equivalent:

(i) The Q_1-completions of $\mathcal{F}_1$ and $\Phi^{-1}\mathcal{F}_2$ coincide.

(ii) The mapping $f_2 \mapsto f_2 \circ \Phi$ from $L^\infty(M_2, \mathcal{F}_2, Q_2)$ to $L^\infty(M_1, \mathcal{F}_1, Q_1)$ is surjective.