

# Probability Theory on Trees and Networks

## Exercise Sheet 13

Submission is due on 02/11/2021 9 p.m.

Please send the solutions to [yannic.broeker@uni-muenster.de](mailto:yannic.broeker@uni-muenster.de) as a pdf-file.

### Exercise 1 (4 points)

Consider a transitive Markov chain  $(X_n)_{n \geq 1}$  with an invariant metric on a graph  $G$  of at most polynomial growth. Suppose that

$$\mathbb{E}_o \left[ \log \left( 1 + d(o, X_1) \right) \right] < \infty,$$

where  $o \in V(G)$  is fixed and  $d(\cdot, \cdot)$  denotes the graph metric. Then show that  $H(X_1) < \infty$  and  $h = 0$ , where  $h$  is the Avez-entropy.

### Exercise 2 (6 points)

Let  $\Gamma$  be a group and  $S$  is a finite and symmetric set of generators. For any probability measure  $\mu$  on  $S$  let  $h(\mu)$  and  $\ell(\mu)$  be the Avez-entropy and the speed of the  $\mu$ -walk on  $\Gamma$ , respectively.

- (i) Show that there exists a symmetric  $\mu$  of maximal Avez-entropy and a symmetric  $\mu$  of maximal speed.
- (ii) Now prove that in general there exists no  $\mu$  that maximizes both. That means, give an example of a group  $\Gamma$  and a set of generators  $S$  such that no probability measure  $\mu$  simultaneously maximizes  $h(\cdot)$  and  $\ell(\cdot)$ .