Seminar on classical and solid locally analytic representations

In this seminar we will study the classical theory, as well as a condensed development, of locally analytic representations. Locally analytic representations of p-adic Lie groups play vital roles for the p-adic Langlands correspondence and its applications. Schneider and Teitelbaum laid both the functional analytic and algebraic foundations for the theory of locally analytic representations in a series of works (e.g., [4, 5]). However, certain inconveniences still remain in the classical theory (for example when considering cohomologies). Condensed mathematics of Scholze and Clausen provides a new approach to deal with vector spaces, algebras, and modules that carry a topology, which particularly include those in the theory of locally analytic representations. Using condensed mathematics, Rodrigues Jacinto and Rodríguez Camargo rebuilt the foundations of locally analytic representation theory in [2].

The first half of this seminar will focus on the classical aspects. Then we will follow [2] to step in the new (and derived) condensed world.

1. **Introduction.** Motivation and overview of the seminar.

2. **Distribution algebra.** This talk follows [4]. Review the concepts and results in functional analysis in [4, §1]. Define the algebra of distribution [4, §2] and prove [4, Thm. 2.2 & Prop. 3.2]. State and explain the duality results [4, Cor. 3.3, Cor. 3.4]. For examples, talk about the distribution algebra when the group is $\mathbb{Z}_p$ [4, §4].

3. **Admissible representations I.** The goal is to define admissible locally analytic representations in [5]. Introduce p-valued pro-p group ([5, §4], see also [3, Ch. V]). For such group G, define Banach algebras $D_r(G, K)$ and $D_{<r}(G, K)$. Introduce Fréchet-Stein algebra and coadmissible modules in [5, §3]. Then state [5, Thm. 4.10, Thm. 5.1] with the proofs postponed to the next talk. Define admissible locally analytic representations and show they form an abelian category [5, Prop. 6.4].

4. **Admissible representations II.** Prove the flatness results [5, Prop. 4.7, Lem. 4.8, Thm. 4.9, Thm. 4.11]. Show the density of locally analytic vectors in admissible Banach representations [5, Thm. 7.1].

5. **Orlik-Strauch representations.** Explain the construction of Orlik-Strauch of some explicit admissible locally analytic representations in [1]. Introduce the category $\mathcal{O}$ and $\mathcal{O}_{\mathrm{alg}}$ [1, §2.5]. Construct the functor $\mathcal{F}_P^G(-,-)$ and present its basis properties [1, Prop. 4.8, 4.9].

Sketch the proof of [1, Prop. 4.2]. State the irreducibility result [1, Thm. 5.3, 5.8]. If possible, say something about the proof of [1, Thm. 5.5], admitting [1, Thm. 5.7].

6. **Non-archimedean functional analysis.** This talk covers [2, §2 & §3], most of which should be already discussed in Oberseminar by the time. Briefly review the materials on solid K-vector spaces and analytic rings [2, §2]. Recall the definitions of various spaces in [2, §3]. The main task is to prove the duality theorem [2, Thm. 3.40].

7. **Solid locally analytic representations I.** Define the distribution algebras in [2, §4.1]. Talk about some explicit description [2, Prop. 4.16]. Define solid G-modules and prove the propositions in [2, §4.2]. Define derived analytic vectors [2, §4.3].

8. **Solid locally analytic representations II.** Finish [2, §4]. Define (derived) analytic and locally analytic representations. Prove the main theorem [2, Thm. 4.36]. State and give proofs for [2, Prop. 4.41, 4.42 & 4.48]. Discuss also [2, Prop. 4.43] if time permits.

9. **Comparison of cohomologies.** Define the cohomologies [2, Def. 5.1] and prove the comparison theorems [2, Thm. 5.3, 5.5]. Explain the ideas of the key lemmas in [2, §5.3]. State the results in [2, §5.5, 5.6].

10. **TBD.** This last talk depends on the progress of the seminar and the interests of the participants. We could discuss the conjectural derived solid aspects of functors like that of Emerton-Jacquet and Orlik-Strauch or choose other topics.

References

[1] Sascha Orlik and Matthias Strauch. On Jordan-Hölder series of some locally analytic representations. *Journal of the American Mathematical Society*, 28(1):99–157, 2015.

[2] Joaquín Rodrigues Jacinto and Juan Esteban Rodríguez Camargo. Solid locally analytic representations of p-adic Lie groups. *Representation Theory of the American Mathematical Society*, 26(31):962–1024, 2022.

[3] Peter Schneider. *p-adic Lie groups*, volume 344. Springer Science & Business Media, 2011.

[4] Peter Schneider and Jeremy Teitelbaum. Locally analytic distributions and p-adic representation theory, with applications to GL_2. *Journal of the American Mathematical Society*, 15(2):443–468, 2002.

[5] Peter Schneider and Jeremy Teitelbaum. Algebras of p-adic distributions and admissible representations. *Inventiones mathematicae*, 153(1):145–196, 2003.