

Oberseminar Topologie: 08.05.2024

Paula Truöl, MPIM Bonn

Title: 3-braid knots with maximal 4-genus

Abstract: This talk in the area of low-dimensional topology deals with the problem of determining the topological 4-genus for the special case of 3-braid knots. The 4-genus of a knot is the minimal genus of a "nicely" embedded surface in the 4-dimensional ball with boundary the given knot. Asking whether a knot has 4-genus zero, i.e. whether it bounds a disk in the 4-ball, is a natural generalization in dimension 4 of the question whether it is isotopic to the trivial knot. It is one of the curiosities of low-dimensional topology that constructions such as finding these disks can sometimes be done in the topological category, but fail to work smoothly. The first examples of this phenomenon followed Freedman's famous work on 4-manifolds. Four decades later, the topological 4-genus of knots - even torus knots - remains difficult to determine. In a joint work with S. Baader, L. Lewark and F. Misev, we classify 3-braid knots whose topological 4-genus is maximal (i.e. equal to their 3-genus). In the talk we will explain the difficulties that arise and draw connections to other problems in low-dimensional topology. We will define all relevant notions and address a broad audience.