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„Old and new results about automorphism spaces and structure spaces of manifolds using tools from L-theory and algebraic K-theory.“

Abstract:

The structure space S(M) of a closed manifold M can be imagined roughly as the space of pairs (f,N) where N is another closed manifold and f:N->M is a homotopy equivalence. It is closely related to the homotopy fiber of the inclusion homeo(M) -> haut(M), where haut(M) denotes the space of the homotopy automorphisms. Namely, that homotopy fiber is a union of connected components of S(M). The surgery theory of the 1960s gives a good understanding of the set of connected components of S(M). This uses the assembly map in L-theory. The surgery theory of the 1960s does not do an equally good job with the higher homotopy groups or the homotopy type of S(M). Hatcher, around 1978, pointed out that a description of the homotopy type of S(M) in a stable range (e.g. homotopy groups up to degree c dim(M), where c is a positive constant independent of M) would also have to use algebraic K-theory. Bruce Williams and I have collaborated on an implementation of this idea for >30 years. I plan to concentrate on the algebraic aspects (algebraic L-theory following Ranicki, and algebraic K-theory following Waldhausen, and how they are related) in the first talk, and more on the geometric aspects (characteristic classes, indices, index theorems) in the second talk.